1. i. An arithmetic progression has first term A and common difference D. The sum of its first two terms is 25 and the sum of its first four terms is 250.

A. Find the values of A and D.

[4]

B. Find the sum of the 21st to 50th terms inclusive of this sequence.

[3]

ii. A geometric progression has first term a and common ratio r, with $r \neq \pm 1$. The sum of its first two terms is 25 and the sum of its first four terms is 250.

Use the formula for the sum of a geometric progression to show that $\frac{r^4 - 1}{r^2 \quad 1} = 10$ and hence

or otherwise find algebraically the possible values of r and the corresponding values of a.

[5]

2. S is the sum to infinity of a geometric progression with first term a and common ratio r.

i. Another geometric progression has first term $2a$ and common ratio r. Express the sum to infinity of this progression in terms of S.

[1]

ii. A third geometric progression has first term a and common ratio r^2. Express, in its simplest form, the sum to infinity of this progression in terms of S and r.

[2]

3. The second term of a geometric progression is 24. The sum to infinity of this progression is 150. Write down two equations in a and r, where a is the first term and r is the common ratio. Solve your equations to find the possible values of a and r.

[5]

4. An arithmetic progression (AP) and a geometric progression (GP) have the same first and fourth terms as each other. The first term of both is 1.5 and the fourth term of both is 12. Calculate the difference between the tenth terms of the AP and the GP.

[5]

5. A geometric series has first term 3. The sum to infinity of the series is 8. [3]
Find the common ratio.

6. The sequence of positive numbers a_1, a_2, a_3, … is a geometric sequence. Prove that the sequence $\ln a_1$, $\ln a_2$, $\ln a_3$, … is an arithmetic sequence. [3]

7. **In this question you must show detailed reasoning.**

A geometric series has first term $(b^2 - 13)$, common ratio $\frac{1}{b}$ and sum to infinity -6.
Find the possible values of the common ratio. [9]

8. Aleela and Baraka are saving to buy a car. Aleela saves £50 in the first month. She increases the amount she saves by £20 each month.

(a) Calculate how much she saves in two years. [2]

Baraka also saves £50 in the first month. The amount he saves each month is 12% more than the amount he saved in the previous month.

(b) Explain why the amounts Baraka saves each month form a geometric sequence. [1]

(c) Determine whether Baraka saves more in two years than Aleela. [3]

9. (See Insert for Jun18 64003.) Consider a geometric sequence in which all the terms are positive real numbers. Show that, for any three consecutive terms of this sequence, the middle one is the geometric mean of the other two. [3]

10. (See Insert for H640/03, Practice 4.)
Assume that the length of each side of the equilateral triangle shown in Fig. C1.1 is one unit.

(a) Find the perimeter of the second iteration, shown in Fig. C1.3. [1]

(b) Find an expression for the perimeter of the *n*th iteration. [3]

(c) Show that, as stated in line 11, the perimeter of the Koch snowflake is not finite. [1]

11. (See Insert for H640/03, Practice 4.)
Assume now that the area of the equilateral triangle shown in Fig. C1.1 is one unit of area.

(a) Find the area of the first iteration, shown in Fig. C1.2. [2]

(b) Find the increase in area between the iterations shown in Figs C1.2 and C1.3. [1]

(c) Find the area of the Koch snowflake. [4]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	(A) $2A + D = 25$ oe	B1		condone lower–case a and d
		i	$4A + 6D = 250$ oe	B1		
		i	$D = 50,$	B1		
		i	$A = -12.5$ oe	B1	**Examiner's Comments** Most candidates formed the correct equations and went on to solve them successfully.	
		i	(B) $\frac{50}{2}\left(2 \times their A + 49 \times their\ D\right)$ [= 60 625] or $\frac{20}{2}\left(2 \times their\ A + 19 \times their\ D\right)$ [= 9250]	M1	or a = their $A + 20D$	
		i	their "$S_{50} - S_{20}$"	M1	$S_{30} = \frac{30}{2}(a + l)$ oe with l = their $A + 49D$	$S_{30} = \frac{30}{2}(2 \times their 987.5 + 29 \times their 50)$
		i	51 375 cao	A1	**Examiner's Comments** Many achieved full marks here. Of those who didn't, most candidates scored two marks for $S_{50} - S_{20}$ with their A and D. A few used S_{21} and just scored 1. Other	

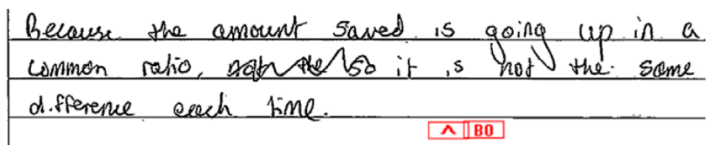
					candidates earned the first mark for u_{21} and about half then earned the second mark for a correct formula with $n = 30$. Fortunately hardly any candidates tried to sum all 30 terms individually.	Geometric Series
		ii	$\frac{a(r^2-1)}{r-1}=25$ or $\frac{a(r^4-1)}{r-1}=250$	B1		
		ii	$\frac{a\frac{(r^4-1)}{r-1}}{a\frac{(r^2-1)}{(r-1)}}=\frac{250}{25}$ oe	M1		allow $a(1 + r)$ as the denominator in the quadruple–decker fraction
		ii	and completion to given result www		at least one correct interim step required	$r^2 = x$ oe may be use
		ii	use of $r^4 - 1 = (r^2 - 1)(r^2 + 1)$ to obtain $r^2 + 1 = 10$ www	M1	or multiplication and rearrangement of quadratic to obtain $r^4 - 10r^2 + 9 = 0$ oe with all three terms on one side	or **M1** for valid alternative algebraic approaches eg using $a(1 + r) = 25$ and $ar^2 + ar^3 = ar^2(1 + r) = 225$
		ii	$r = \pm 3$	A1		or **B2** for all four values correct, **B1** for both r values or both a values or one pair of correct values if second **M** mark not earned
		ii	$a = 6.25$ or -12.5 oe	A1	or **A1** for one correct pair of values of r and a **Examiner's Comments** Most earned the first mark, but then there was much toil for the second mark, which was often not earned due to wrong working or to leaving too much to the marker's imagination. Faced with solving the given statement, most opted for multiplying by r^2 -1 and were then stumped by the quartic. Careless work led to $r^2 = 10$ or 11. A good number of candidates who successfully found r neglected to find a. A small number of candidates produced elegant work for full marks.	
			Total	12		
2		i	$2S$ cao	B1	<u>Examiner's Comments</u>	

					Most candidates did not earn this mark: in spite of $\frac{2a}{1-r}$ being commonly seen, candidates were unable to make the connection to "S". Those who did, often left their answer embedded in irrelevant working.	Geometric Series
		ii	$\frac{a}{1-r^2}$	M1	if **M0, SC1** for $\frac{1-r}{1-r^2} \times S$ oe <u>Examiner's Comments</u> Approximately three quarters of candidates made the correct initial move of $\frac{a}{1-r^2}$. A few then recognised that factorising the denominator was relevant, but only a tiny minority went on to earn the second mark.	
		ii	$\frac{S}{1+r}$ or $\frac{1}{1+r}S$	A1		
			Total	3		
3			$ar = 24$ (i)	B1*		allow $ar^{2-1} = 24$
			$\frac{a}{1-r} = 150$ (ii)	B1*		
			correct substitution to eliminate one unknown	M1dep*	eg subst. of $a = 150(1-r)$ or $r = \frac{150-a}{150}$ in (i) alternatively, subst. of $a = \frac{24}{r}$ or $r = \frac{24}{a}$ n (ii)	if M0, B1 for both values of r and B1 for both values of a, or B1 for each pair of correct values NB $150r^2 - 150r + 24$ [= 0] $a^2 - 150a + 3600$ [= 0]
			$r = 0.8$ or 0.2	A1	or A1 for each correct pair of values	A0 if wrongly attributed

			$a = 30$ or $a = 120$	A1	ignore incorrect pairing if correct values already correctly attributed **Examiner's Comments** Most candidates wrote down the required equations, and most went on to eliminate one of the variables correctly. What followed often proved too difficult, and no further marks were earned. A number of candidates obtained negative answers for both *a* and *r*, but never suspected anything was amiss.	A0 if wrongly attributed
			Total	5		
4			$1.5 + (4 - 1)d = 12$ or better	M1	or $1.5 \times r^{(4-1)} = 12$ or better	if first **M0 B0** allow **B3** for $d = 3.5$ **and** $r = 2$;
			$d = 3.5$	A1	$r = 2$	**B2** for one of these; may be embedded in calculation of difference
			$r = 2$	B1	$d = 3.5$	
			$1.5 \times$ their $2^9 - (1.5 + 9 \times$ their $3.5)$ oe	M1	**M0** for use of their S_{10} in either term	**NB** 768 − 33
			differences = 735	A1	**Examiner's Comments** This was done very well indeed, with many candidates scoring full marks. A few slipped up with the arithmetic and lost the accuracy marks, but the method was very well understood.	allow −735
			Total	5		
5			$\frac{3}{1-r} = 8$ $\Rightarrow 3 = 8(1 - r)$ $\Rightarrow r = \frac{5}{8}$	M1(AO1.1) M1(AO1.1) A1(AO1.1) [3]	Use of correct formula Clearing fraction	
			Total	3		

6			Sequence of the form $\ln a$, $\ln ar$, $\ln ar^2$ … $\ln a$, $\ln a + \ln r$, $\ln a + 2\ln r$… Arithmetic sequence with common difference $\ln r$	M1(AO 2.5) M1(AO 3.1a) E1(AO 2.2a) [3]	Use of notation for geometric sequence Laws of indices used Laws of indices used	
			Total	3		
7			DR $\frac{b^2-13}{1-\frac{1}{b}}=-6$ $\frac{b(b^2-13)}{b-1}=-6$ $b(b^2-13) = -6(b-1)$ $b^3 - 7b - 6 = 0$ $(-1)^3 - 7 \times (-1) - 6 = 0$ $(b+1)(b^2 - b - 6) = 0$	M1(AO 3.1a) M1(AO 3.1a) A1(AO 1.1) M1(AO 1.1) M1(AO 2.1) A1(AO 1.1)	Use of sum to infinity to form an equation Starting to clear fractions Correct equation without fractions Cubic with zero on one side Use of factor theorem to search for factor Correct fact	

			$(b+1)(b+2)(b-3)=0$ Roots -1, -2, 3 -1 cannot be the common ratio of a geometric sequence with a sum to infinity Possible common ratios are $-\frac{1}{2}$ and $\frac{1}{3}$	M1(AO 1.1) B1(AO 2.3) A1FT(AO 3.2a) [9]	Method for solving quadratic Rejection of root that does not make sense in the context Follow through their values of *b*	Geometric Series
			Total	**9**		
8		a	Arithmetic sequence with $a = 50$, $d = 20$ $S_{24} = \frac{24}{2}\left(2\times 50+\left(24-1\right)20\right)$ = £6720	M1 (AO 1.1a) A1 (AO 1.1b) [2]	Using appropriate formula for sum of an arithmetic sequence with $a = 50$, $d = 20$ Allow full credit for any correct method Allow for total written out in full <u>Examiner's Comments</u> Some candidates used a brute force method, writing out the complete list of monthly payments and adding them. Most successfully identified this as an arithmetic series and used the correct formula to find the sum of 24 terms.	

					Partial credit was not awarded where a candidate found the 24th term but did not then attempt to find the sum total.	
		b	Each month is 12% more than the previous, so multiplied by 1.12 giving a geometric sequence with $a = 50$, $r = 1.12$	E1 (AO 2.4) [1]	Clear argument must include the value 1.12 **Examiner's Comments** Many candidates realised that the 12% increase could be achieved by multiplying by 1.12 which leads to a geometric series. The value 1.12 had to be seen in part (b) for the mark to be credited. Exemplar 2  Some candidates were not credited the mark as their answer was too vague, as in the exemplar above. AfL Even if 1.12 was used in part (c) this mark could not be credited if 1.12 was not seen in part (b).	
		c	Geometric sequence with $a = 50$, r = 1.12 $S_{24} = \frac{50(1.12^{24} - 1)}{0.12}$	M1 (AO 3.1a)	Using appropriate	

			= £5907.76 which is less than Aleela	A1 (AO 1.1b) E1 (AO 2.1) [3]	formula for sum of a geometric sequence with $a = 50$, $r = 1.12$ | Allow for total written out in full Allow any suitable rounding FT their values (dep on earning the M marks in part (a) and (c)) Examiner's Comments Again the formula for the total of terms was needed to earn the method mark. The final mark was credited to candidates who compared their totals and follow-through was available to those who had made an arithmetic mistake but not to those who had not earned the method marks in both part (a) and part (c).	
			Total	6		
9			Let the terms be $\frac{c}{r}$, c, cr The geometric mean of first and last is $\sqrt{\frac{c}{r}cr}$	B1 (AO 3.1a) M1 (AO 1.1) E1 (AO 2.1)	Expressions for three consecutive terms of a GP (any correct form) Expression for GM of first and last term (any correct form) FT their terms **AG** Correct completion	

			$\sqrt{\frac{c}{r}cr} = \sqrt{c^2} = c$; this is the middle term	[3]	**Examiner's Comments** Most candidates were able to give general expressions for three consecutive terms of a geometric series and their response progressed successfully. However, some candidates lost a mark by not correctly completing their explanation by relating to the middle term.	
			Total	3		
10		a	$\frac{16}{3}$ [units]	B1 (AO 2.2a) [1]	oe	
		b	At each iteration, the perimeter is $\frac{4}{3}$ times the previous perimeter Perimeter of *n*th iteration $= 3\times\left(\frac{4}{3}\right)^n$	M1 (AO 3.1a) A2 (AO 1.1, 2.2a) [1]	May be a specific instance Allow **A1** for 3 times a power of $\frac{4}{3}$	
		c	As $n \to \infty,\ 3\times\left(\frac{4}{3}\right)^n \to \infty$	E1 (AO 2.4) [1]	**or** $3\times\left(\frac{4}{3}\right)^n$ increases without limit as *n* increases	
			Total	5		
11		a	Area of each little triangle $= \frac{1}{9}$ unit	M1 (AO 2.2a) A1 (AO	**or** total additional area $= \frac{1}{3}$	

			Area $= 1\frac{1}{3}$ [units]	1.1) [1]		Geometric Series
		b	$\frac{12}{81}$	B1 (AO 2.2a) [1]	oe $\frac{4}{27}$	
		c	$1+\frac{3}{9}+\frac{4\times3}{9\times9}+\frac{4\times4\times3}{9\times9\times9}+K$ $1+\frac{\frac{3}{9}}{1-\frac{4}{9}}$ $\frac{8}{5}$ [units]	M1 (AO 3.1a) M1 (AO 2.2a) M1 (AO 3.1a) A1 (AO 1.1) [5]	4 times for new number of triangles Triangle area divided by 9 Use of GP formula	
			Total	7		